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The 4-Percent Rule Was Never Failproof: On the Folly of Fixed Rate Withdrawals

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ON THE FOLLY OF FIXED RATE WITHDRAWALS

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Abstract

The 4-percent withdrawal rule was formulated by William Bengen to satisfy a specific historical test. Bengen sought to determine the initial rate which, applied against a balanced portfolio of stocks and bonds, and adjusted annually for inflation, always had sustained withdrawals for at least 30 years over the span of available data. This paper probes the flaws in Bengen's U.S. historical data that led to the mistaken nomination of 4 percent as a safe withdrawal rate. After documenting the historical failures of the 4-percent rate, the paper develops the conditions that determine whether any fixed rate of withdrawal can be sustained and assesses the impact of more- versus less-conservative asset allocations. The conclusion is pessimistic: fixed-rate withdrawals tested on history are inherently flawed.

The 4-percent rule long ago fell from favor as anything but a rough guide. Historical data on international markets gathered by Pfau (2010) and Anarkulova et al. (2022) showed that a withdrawal rate that high could not have been sustained in markets outside the United States. Bengen's (1994) data for U.S. stocks and bonds during 1926–1992 emerged as a special case—his findings did not generalize to other times and places. Once that failure was recognized, emphasis shifted to the development of flexible schemes for withdrawal, as in the work of Blanchett (2007).[1]

However, outside of academia the 4-percent rule remains alive and well. It certainly continues to inform financial journalism. Discussions querying whether the 4-percent rate might be about to fail for the first time in the United States (Finke et al. 2013), or that the rate might need to be lowered to reflect the changed circumstances of the economy following the Great Financial Crisis, continue to be published. Practitioners can be forgiven for supposing that 4 percent must still be a good starting point, because, after all, that rate did succeed in Bengen's data.

This paper presents new historical data showing that 4-percent withdrawals could not have been sustained by investors in the field, not in the United States and not within Bengen's period. The paper then proceeds to an examination of the conditions under which any fixed withdrawal rate will succeed or fail. It also considers the role of asset allocation in sustaining withdrawals. The conclusion is pessimistic. When 60/40 fails, 30/70 and similar more-conservative allocations also are likely to fail. Asset allocation provides no panacea when it comes to sustaining fixed withdrawals under challenging circumstances.

What Bengen Did

Bengen (1994) focused on balanced portfolios of stocks and bonds. A 50/50 split was used to support the 4-percent rate (other allocations were studied, but the 50/50 split was made the central case for exposition). Second, Bengen set 30 years as the criterion period. He made no claim that 4 percent would work over longer horizons, although he did report numerous cases where 50 years could have been sustained at that rate. Third, stocks for Bengen meant the S&P indexes as tabled in Ibbotson's (2020) *Stocks, Bonds, Bills & Inflation Yearbook*, or SBBI, using total returns based on monthly reinvestment of dividends. Bonds meant five-year Treasury bonds, also as tabled in the SBBI, with monthly reinvestment of interest income and an annual swap out of the now four-year bond for a new five-year bond.[2] When Bengen wrote, the available SBBI data stopped at the end of 1992; in order to include 30-year periods with beginning dates through 1976, Bengen extrapolated past returns on these two assets out to future years.[3]

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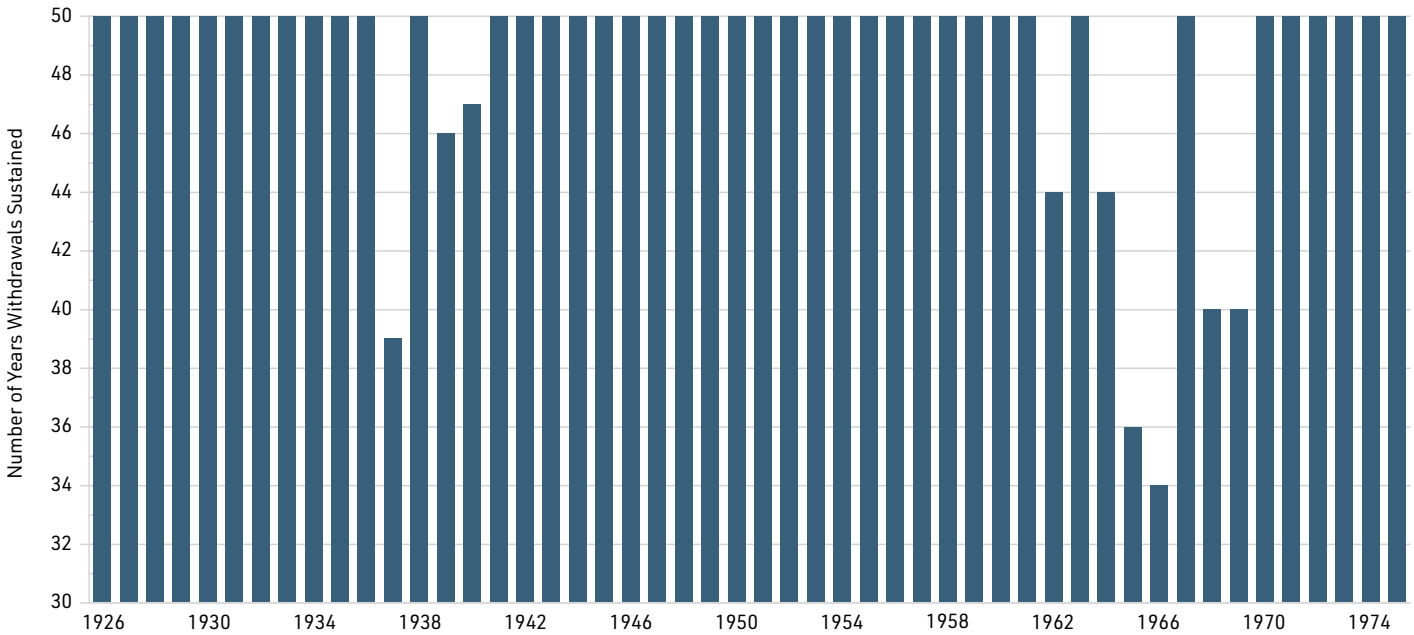
Finally, and perhaps uniquely in the withdrawal literature, Bengen takes the first withdrawal at the end of the first 12 months; and that withdrawal is not 4 percent of the end-of-year balance but 4 percent multiplied by the inflation that occurred during that year, i.e., if inflation had been 3 percent, then 4.12 percent gets withdrawn.[4] What funds the retiree used to cover living expenses during those first 12 months of retirement is not stated.

Using fresh calculations from updated SBBI data and actual returns from 1993 forward, figure 1 is a representation of the Bengen (1994) base case, the 50/50 split with a 4-percent withdrawal rate (modeled on

FIGURE 1 How the 4-Percent Rule Fared in Bengen's Data



Note: Bengen (1994) did not have data after 1992 and made a straight-line projection of future stock and bond returns. Therefore figure 1, based on actual data through 2024, does not agree in detail with Bengen (1994, figure 1b) for results after 1961. For example, although 1966 is again the low point, the actual data show that 4-percent withdrawals could have been sustained for 34 years, not the 33 years that Bengen projected.

Source: Author's calculations.

his figure 1b). Thirty years proved a forgiving criterion. In many cases inflation-adjusted 4-percent withdrawals could have been sustained for 50 years.[5]

External Validity of Bengen's Analysis

External validity concerns the applicability of findings obtained in an artificial setting. A laboratory study of risk aversion, for instance, might use college sophomores. Written descriptions of several possible investments will be given, with subjects asked to indicate their preferred option. The experimenters will have structured these options to index varying degrees of some theoretical property, such as uncertainty. The experiment is run, the data are analyzed, and the hypotheses that guided the experiment are reported as supported or rejected.

External validity refers to whether these results obtained in the laboratory—using college students as subjects, written descriptions rather than actual investments, and assumed funds rather than real money—will generalize to prospective retirees who must choose among the heterogenous investments offered in today's marketplace, and who have $1 million that they need to have last through retirement.

Bengen (1994) was in effect a laboratory study. Investment returns were taken straight from the SBBI appendix. When the five-year Treasury was rolled forward each year, it was not necessary to sell it at the bid price or to buy the next one at the ask price, nor to pay any commission on either transaction. Likewise, the semi-annual coupon, which in some years was as low as $7.50 on a $1,000 bond, was divided into six tranches of as little as $1.25 and each of these was invested without transaction cost into a $1,000 bond each month. Similarly, the monthly dividend income derived from quarterly payments by the 500 S&P stocks, call it $1,100 per month on a $500,000 stock portfolio,[6] would be reinvested without commission, in proportion to capitalization, with perhaps $50 of that dividend income going into the largest single stock in the index and five pennies into the smallest.

Investors outside the laboratory could not have invested this way. They would either have paid ruinous commission costs—if a stockbroker would even have accepted a five-penny transaction for a fractional share—or they would have had to pay the expenses of a mutual fund to make these investments for them. Either way, calculations based strictly on values taken from the SBBI appendixes lack external validity.

Investor Returns Outside the Laboratory

The SBBI still was new when Bengen wrote. For example, the five-year Treasury series had just been added in 1989. Nothing like the SBBI had existed before Ibbotson and Sinquefield (1976). Bengen's historical analysis was at the leading edge in 1994—neither Ibbotson nor anyone else had considered the application of withdrawal rates against the new SBBI series.

In the intervening 30 years historical datasets have come a long way. Pfau (2010) could draw on the efforts of Dimson et al. (2002/2025) to examine returns in 17 international markets back to 1900. Anarkulova et al. (2022) could draw on global financial data for international returns on several dozen markets back into the nineteenth century.

All these datasets typically make the same assumptions as the SBBI: Cost-free and frictionless transactions can be made in any amount at any frequency, whether to buy, sell, rebalance, or reinvest interest and dividends. Jones (2002) documents the magnitude of these omitted costs, showing that all-in transaction costs peaked in the 1960s and 1970s at 100–200 basis points (bps) per stock transaction.

The present paper takes advantage of a newly assembled dataset that does include costs. McQuarrie (2023) compiled annual returns on the largest U.S. mutual funds, anchoring the beginning of the series in December 1925 and carrying returns through 1986, after which index mutual funds were substituted through 2023. The series thus spans the same period as the SBBI series used by Bengen (1994) with an extension to the present. Even by the 1930s the largest mutual funds had more than $100 million in assets, sufficient to reinvest dividends in proportion to capitalization and large enough to pay the lowest commission costs on offer. For purposes of rebalancing, withdrawals without charge have been a defining feature of open-end mutual funds since their advent in 1924 (Bullock 1959; Fink 2011).

There can be no question of external validity with these mutual fund returns. Following passage of the Investment Company Act of 1940, returns must by law be reported after all costs. The historical returns of mutual funds thus reflect what investors could have received outside the laboratory. The next section shows how the 4-percent rule would have failed these investors.

Failure of the 4-Percent Rule Using Balanced Funds

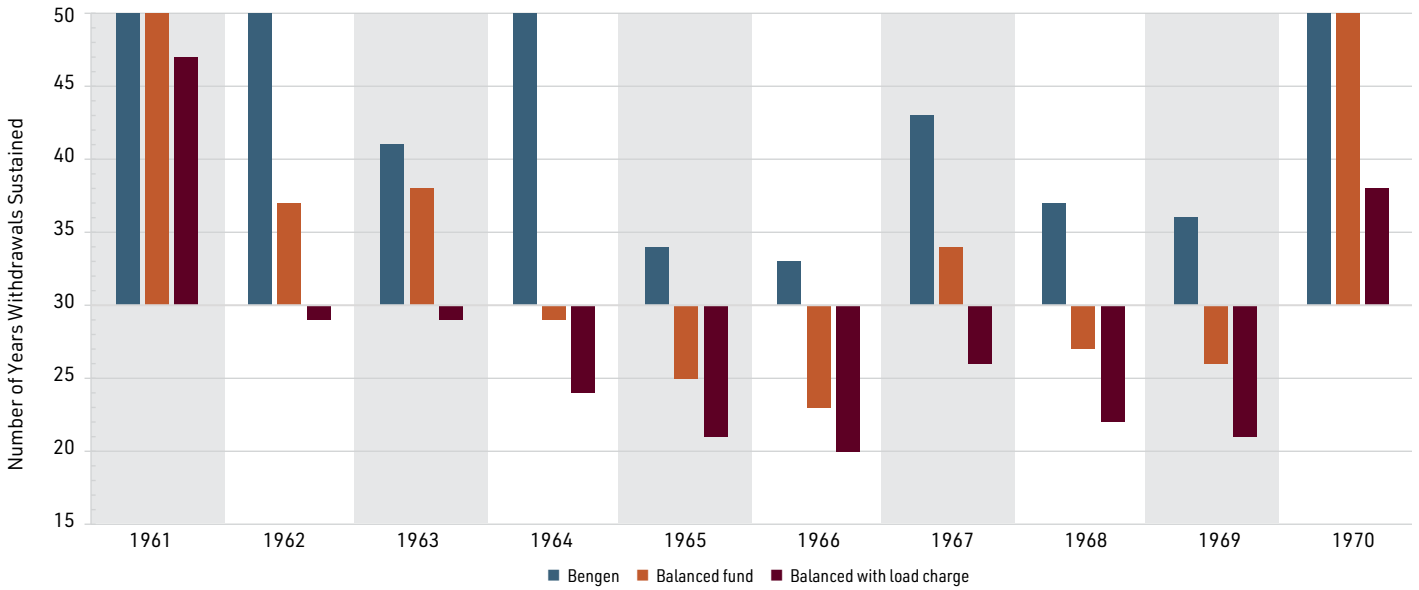
McQuarrie (2023) constructed two mutual fund series: one for common stock funds and one for balanced funds. The latter started with a 50/50 stock bond allocation in the 1930s (Moffitt 1952), moving in the 1950s toward a 60/40 allocation.[7] The balanced fund index thus parallels Bengen's base case and is examined first. As a one-stop investment, the balanced fund requires no manual rebalancing by the investor, and by the 1960s, automatic reinvestment of dividends and interest could be elected by the fund investor, adding further validity. Other fund series will be introduced later when the conditions for failure of any fixed withdrawal rate are probed.

Figure 1, styled after Bengen, showed that his 4-percent rule was threatened at only two junctures: in the late 1930s and in the 1960s. Everywhere else, it would have sustained withdrawals for more than 50 years in the laboratory data. Preliminary tests with balanced funds showed that a $1-million portfolio decumulating by the 4-percent rule would have just (barely) survived the severe downturn in 1937, when the SBBI shows a stock market decline of 35 percent.[8]

Accordingly, figure 2 shows test results for the 1960s only. Two levels of external validity were considered. In the 1960s all large balanced mutual funds carried sales loads, and these loads had been present from the outset in the 1920s. The database contains the weighted average load, around 7 percent or somewhat more in this period, and this was first subtracted from the $10,000 initial investment to get the results charted by the maroon bars in figure 2. These results are valid for an investor who made a lump-sum investment upon retirement in the 1960s.

On the other hand, all load funds discounted their sales charges for larger investments, down to only 2–3 percent for amounts of more than $100,000. In addition, a small investor who accumulated an investment

FIGURE 2 Failure of the 4-Percent Rule in the 1960s Outside the Laboratory



Source: Author's calculations.

over decades would not have paid the load on any in-fund appreciation, therefore subtracting the full load is problematic. Finally, load funds are not common today. The orange bars show the results without any sales load.

As figure 2 shows, for start dates in the 1960s, the 4-percent rule would have failed investors in the field year after year. With a sales load, the U.S. investor would have run out of money for every start date from 1962 to 1969 inclusive. Ignoring the sales load, the 4-percent rule would have failed in five cases. The very worst start date was 1966. Without a load, a 65-year-old retiree who started retirement that year would have run out of money at age 88; with a load, funds would have been exhausted by age 85.

Reasons for Failure

Failure of the 4-percent rule for U.S. investors outside the laboratory prompts the following questions:

1. What was distinctive about asset returns in the 1960s? Why did failures cluster for start dates here and nowhere else?
2. How could two adjacent start years, such as 1969 and 1970, produce such discrepant results when their respective asset return sequences are identical except for a shift of one year?
3. Could a different asset allocation have prevented or at least ameliorated the failures seen in the 1960s, relative to the approximately 60/40 allocation used by balanced funds of that era?

There are 50 start dates in figure 1. Using the conservative approach of ignoring sales loads, but using actual mutual funds, the 4-percent rule would have succeeded in 45 cases and failed in five. The failures are nonetheless failures of moment; the 4-percent rule was supposed to be failproof in the U.S. historical record to this point. Telling retirees who run out of money in their 80s, after following the 4-percent rule, that the odds had been nine out of 10 in their favor is not likely to mollify them.

Nonetheless, a sequence of asset returns rare enough to occur in one out of 10 cases must be distinctive in some respect. Consider that retiring in January 1930—three months into the stock market crash of 1929, after stocks had fallen 25 percent with another 75-percent decline still to come—did not lead to failure for investors with a balanced portfolio, neither in Bengen's laboratory data nor in balanced funds with a load. Likewise, retiring in 1970, just before the great inflation of the 1970s, did not lead to failure.[9]

These examples show that a fixed withdrawal rate, such as 4-percent real, is not hostage to a sudden steep market decline and can withstand quite high rates of inflation that continue for a decade or more after retirement. To suffer the outcomes experienced by investors who retired in the 1960s requires a more specific set of circumstances.

... IN THE 1960S, THE 4-PERCENT RULE WOULD HAVE FAILED INVESTORS IN THE FIELD YEAR AFTER YEAR. WITH A SALES LOAD, THE U.S. INVESTOR WOULD HAVE RUN OUT OF MONEY FOR EVERY START DATE FROM 1962 TO 1969 INCLUSIVE.

For a fixed rate of withdrawal to fail, as the 4-percent rule did in the 1960s, the sequence of returns must show: (1) low real returns on the portfolio, (2) commencing near the start date, and (3) continued for long enough.

Retirement after 1929 was not a problem because the decline was soon reversed and, during its brief course, was ameliorated by severe deflation. Retirement in 1970 was not a problem because, after the 1973–1974 bear market, and despite high inflation, stocks resumed earning real returns and, somewhat later, bonds did likewise.

The next section recasts these preconditions for failure in more general terms.

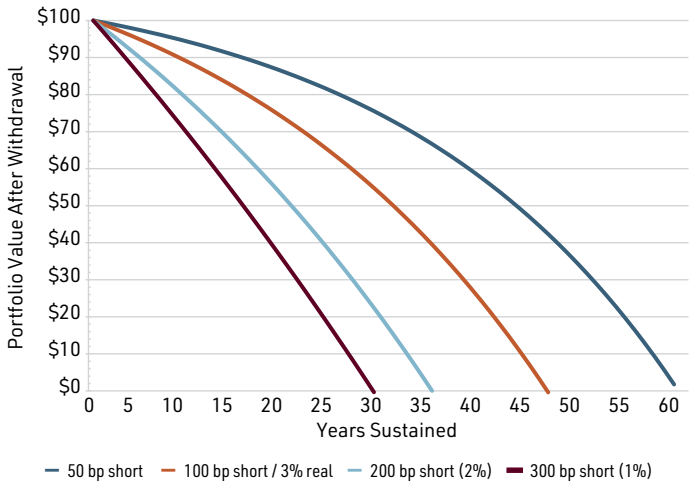
Tipping Point Phenomena

Any fixed rate of withdrawal, applied against an exponential series (compounding returns) and adjusted for inflation, itself an exponential series, is subject to exponential exhaustion. If returns fall far enough short of some level, for long enough, the remaining balance must go to zero before 30 years have passed, failing the Bengen test. To use a familiar example, exponential exhaustion explains why a mortgage can be paid off on a finite amortization schedule. A monthly payment just a few dollars more than the initial monthly interest cost leads inexorably to a zero balance 360 months later.

Taking withdrawals from a portfolio of assets is more complicated, but the principle is the same. If the retiree had available a perpetuity, i.e., an annuity that pays out indefinitely, paying a real return of 4.0 percent each year, then inflation-adjusted 4-percent withdrawals, made at the end of each year, could be continued indefinitely.[10] But a withdrawal rate any higher than 4.0 percent eventually would exhaust the funds, parallel to how amortization of a mortgage proceeds.

Next, consider the case where the rate paid on the perpetuity is less than 4.0 percent but withdrawals of 4 percent are taken. Figure 3 shows the impact of varying degrees of shortfall on how long 4-percent withdrawals can be sustained from that perpetuity. If the rate of return falls just 50 bps short, at 3.50 percent, the period that 4-percent withdrawals can be sustained contracts by a vast amount but still extends 60 years. A more significant shortfall of 100 bps contracts the period further but still sustains withdrawals for longer than most retirements last. At a 200-bps shortfall, i.e., a real return of 2.0 percent against withdrawals at 4 percent, withdrawals begin to approach failure, sustaining only 35 years. A shortfall of just 100 bps more produces failure, and returns on the instrument, now at 1-percent real, lead to exhaustion in 29 years.

FIGURE 3 Idealized Account of How Shortfalls in Real Return Can Lead to Premature Exhaustion



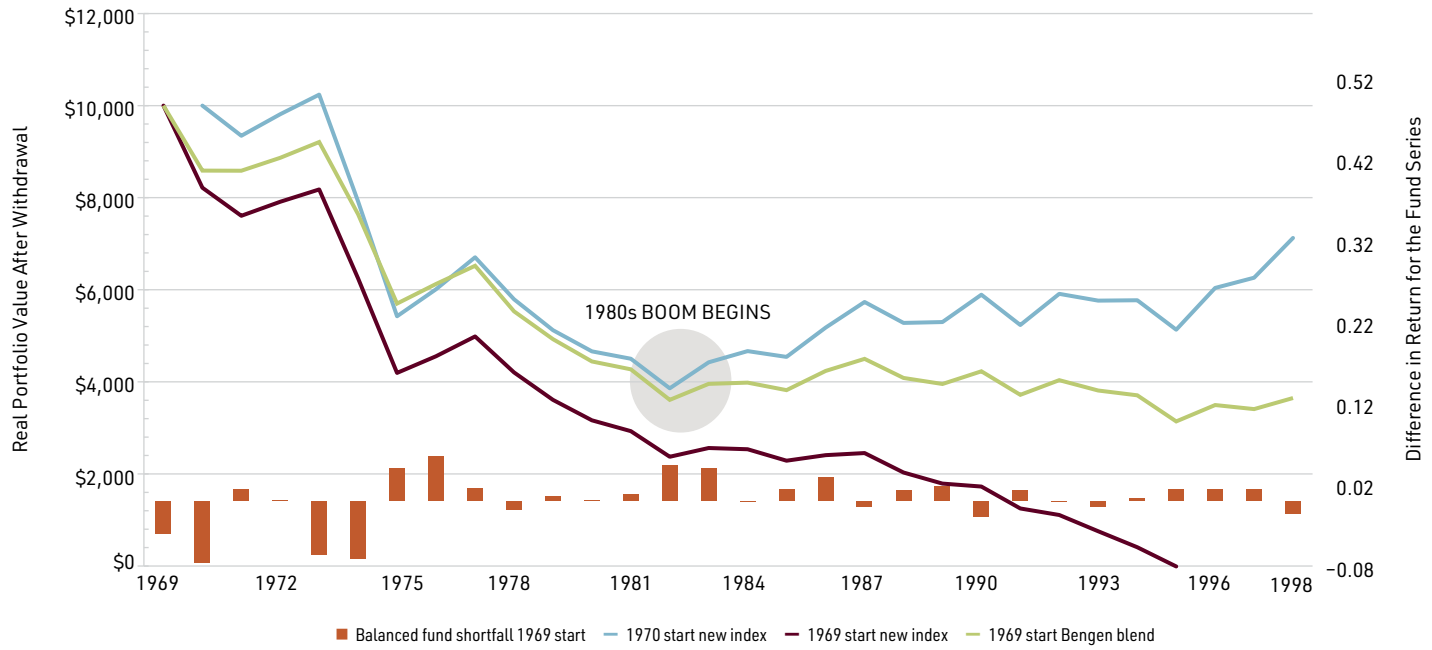
Source: Author's calculations.

And that is why the 4-percent rule succeeded in Bengen's laboratory data during the 1960s, even as it would have failed using actual mutual fund returns from that era. Unbeknownst to Bengen or his readers, the real return on a balanced portfolio during the 1960s was very close to the tipping point. The small difference in return pattern for the mutual fund series relative to the cost-free SBBI series was sufficient to cause the 4-percent rule to fail.

The theoretical examples so far omit two important features of portfolio performance. First, asset returns fluctuate; and second, these fluctuations are ordered in time. The latter property is important because the arithmetic operation of subtraction followed by multiplication is not commutative—the order of these operations matters. Subtracting a fixed (real) amount when lower returns are ordered first will not sustain withdrawals for the same number of years as if those lower returns had come later in the sequence.[11] Even if the average return on the portfolio appears adequate, i.e., does not fall short as illustrated in figure 3, if the variability is great enough, or the sequence is unfortunately arranged, premature exponential exhaustion still may occur.

Figure 4 illustrates further how premature exponential exhaustion hinges on a tipping point. The blue line shows the new balanced fund results for retirement in 1970. This portfolio still was going strong after sustaining 4-percent withdrawals for 30 years. The maroon line shows the new balanced fund results for retirement one year earlier, in 1969.

FIGURE 4 The Tipping Point as Seen in Retirements in 1969 or 1970



Source: Author's calculations.

The 4-percent rule exhausted this portfolio at the end of 25 years, per figure 2. The light green line uses Bengen's laboratory returns for a 1969 retirement; this portfolio sustained withdrawals for 39 years, comfortably in excess of the 30-year criterion. The oval marks the end of 1981, when the great stock and bond boom of the 1980s and 1990s began.

Taking the 1969 comparison first, the orange bars at the bottom of figure 4 show the excess or shortfall of return for the new balanced fund index relative to the Bengen blend. During the 30 years through 1998, the balanced fund index average return (arithmetic) outperformed the Bengen blend by a slight 0.15 percent per year. But the four cases of substantial underperformance—annual shortfalls of 4 percent to 8 percent in returns—all occurred in the first six years. By the time the markets turned after 1981, the Bengen portfolio had given up 64 percent of its starting value in real terms. The ongoing $400 of real withdrawals represented an 11-percent withdrawal rate from the portfolio value at that point. The balanced fund portfolio, however, had given up 76 percent of its real starting value. The ongoing real $400 of withdrawal represented an almost 17-percent withdrawal rate on what remained.

The great boom of the 1980s and 1990s was sufficient to fund the 10–11-percent withdrawals against the Bengen portfolio for years to come. But not even that boom could fund the withdrawals of 17–20 percent that had to be taken against the balanced fund portfolio. It had passed the tipping point, 13 years in, before the boom commenced.

Next, the blue and maroon lines allow a comparison of the effects of timing—retiring in 1970 versus one year earlier in 1969—using the new mutual fund data. Except for the first-year return, the two retirees receive identical returns with the blue and maroon lines exactly parallel through the tipping point. However, the 1969 retiree lost 8.58 percent nominal in 1969, which—given the 6-percent inflation of that year—represented a real return of negative 14 percent, from which $424, reflecting that same 6-percent inflation, had to be subtracted. After one year, the 1969 retiree had thus suffered a significant impairment of wealth, with only $8,200 remaining in real terms, relative to the 1970 retiree, who starts out that year with $10,000. As was the case with the Bengen portfolio, the 1970 retiree reached 1981 down only about 60 percent; the 1970 retiree's nominal withdrawal always is 6 percent less than that of the 1969 retiree from having skipped 1969's inflation. Therefore the 1970 retiree escapes the tipping point.

Based on these field comparisons, a return to the laboratory may be useful to further elaborate the elements that drive premature exponential exhaustion. Consider Bengen's return series for retirement in 1966, the worst case in his analysis, same as this paper, with withdrawals sustained for only 34 years. Three manipulations to the SBBI return series will be considered in an effort to precipitate failure of the 4-percent rule.

To begin, a constant was subtracted from each of the 30 annual returns from 1966 to 1995.[12] This reduces the real return but leaves the standard deviation and sequence of returns unaltered. A subtraction of 33 bps was found necessary to produce exhaustion at 29 years and 10 months. For context, the real annualized return for those 30 years had been 4.18 percent per the SBBI; after the subtraction it was 3.86 percent.[13] That is why the planner can have no confidence in fixed withdrawal rates—it is not possible to rule out the possibility of a future return series averaging just 33 bps lower than that of Bengen's 1966–1995 laboratory series. Put another way, had Bengen applied a fee to his laboratory returns of even 0.50 percent of assets under management (AUM), he would have had to report failure of the 4-percent rule in the 1960s for clients who used such an advisor.[14]

As a second test, a constant was added to the 15 highest returns in that period and the same constant was subtracted from the 15 lowest returns. This makes the series more volatile without changing the arithmetic mean or the sequence. An addition (subtraction) of 2.0 percent was sufficient to induce failure at about 29.7 years, versus 34 before the alteration. The standard deviation of the Bengen blend before alteration had been 9.99 percent; after the additions and subtractions, it expanded to 11.66 percent, and this sufficed to reach the tipping point.

For the sequence alteration the worst annual return, for 1974, was moved up to be the first-year return and the 1966–1973 return sequence was shifted forward to fill the gap. That one change caused the count of years sustained to drop from 34 to just under 29. If the worst annual inflation, for 1979, also was moved up to 1966, then the count of years sustained dropped further, to 26. These small changes to the sequence of returns do not affect the arithmetic mean or the variance but nonetheless had a powerful impact on the years sustained.[15]

In summary, the success of any fixed withdrawal rate, such as 4 percent in Bengen's laboratory study, is quite fragile. A small reduction in return of a few dozen basis points can turn success into failure. Given two series with identical arithmetic mean returns, if one is made to have a standard deviation on the order of 12 percent instead of 10 percent, that small alteration can produce failure in the more volatile case. A slight but unfavorable reordering of the return sequence can do the same. As acknowledged at the outset, this is why the thought leaders in the planning community have moved toward flexible systems for determining withdrawal rates. This is also why a fixed rate that happened to succeed across decades of market history, arbitrarily sampled from one market and in one century (the United States in the 20th century), can give little confidence of continued success.

Could Asset Allocation Have Rescued the 4-Percent Rule?

The balanced mutual funds used in the new series were not index funds with a commitment to a specified mix of stocks and bonds. The exact mix varied over time within funds and across funds. Wellington, one of the largest such funds, notoriously engaged in poor market timing around 1970, as later recounted by John Bogle (2019). Any of these actively managed funds at any point could have made poor decisions about the amount allocated to stocks, the selection of stocks, the credit quality of the bonds selected, or their duration. In that sense, the analyses reported so far have no internal validity as a test of whether a specified mix of

stocks and bonds could always have sustained 4-percent withdrawals in the United States.

Fortunately, the new mutual funds database contains a separate stock fund series and fixed income series. These can be assembled into any desired mix with rebalancing to maintain that mix. This section considers two allocations of note. The 60/40 mix has anchored balanced fund allocations since Peter Bernstein (2002) extolled it. More recently, the 30/70 mix was selected by Vanguard for its retirement-income fund. All of Vanguard's target-date retirement funds are converted into the retirement-income fund seven years after the target date is reached. The fund assumes that investors, who likely will be taking required minimum distributions by that point, will hold the fund with that 30/70 mix for the remainder of their lives.[16] Put another way, the largest provider of target-date retirement funds, which by legislation are now the default investment option in workplace retirement plans, has opted for a 30/70 allocation as the single best and most widely applicable asset allocation for retirees withdrawing funds.

The historical analysis that follows considers these two mixes along with the two pure options: all stock and all fixed income. Conventional wisdom holds that the volatility of an all-stock portfolio, once decumulation begins at retirement, too often will lead to failure in the form of premature exponential exhaustion. Conventional wisdom also holds that a 100-percent allocation to fixed income will not give enough return to withstand bouts of inflation. But there is no settled view on whether a 30/70 or 60/40 allocation offers more robust support for a fixed withdrawal rate such as 4 percent across the variety of market scenarios that have unfolded historically.

Many retirees suppose that their portfolios will be safer with half as much in stocks—30 percent versus 60 percent. This probably is true if risk is defined narrowly in terms of standard deviation. But it is an empirical question whether a conservative or moderate allocation would have fared any better during the crucible of retirement in the 1960s—or during the quite different vicissitudes of retirement in the 1930s. It also is of interest to see just how badly a 100-percent stock portfolio would have fared compared with putting everything in fixed income. During the past three to four decades, most authorities have expected that, for fortunately timed retirements, the upside will be greater with a greater allocation to stocks, i.e., more wealth will be available for legacy purposes. The problem, of course, concerns what happens when the timing of retirement is unfortunate. Did a 60/40 portfolio better sustain 4-percent withdrawals relative to an all-stock portfolio across fortunate and unfortunate retirement starts? Or was it necessary to ratchet stock exposure down to 30 percent to promote success? Or did that conservative portfolio sometimes fail?

Were none of the allocations to succeed in sustaining 4-percent withdrawals, every year, across these difficult periods, that would add to the evidence that no fixed withdrawal rate can be safe.

Asset Allocation and the 4-Percent Rule

Four allocations were considered:

- A 100-percent allocation to a portfolio of large stock mutual funds, as compiled in McQuarrie (2023).
- A 60-percent allocation to the same portfolio of large stock mutual funds and 40-percent to a savings deposit.
- A 30-percent allocation to the same portfolio of large stock mutual funds and 70-percent to a savings deposit.
- A 100-percent allocation to a savings deposit.[17]

Sales loads are ignored, because these would not apply to the savings deposit, thus confounding the effects of stock allocation and load. McQuarrie (2023) used a savings deposit as the fixed income option, again for purposes of external validity: The ordinary investor could not have rebalanced small amounts into or out of a Treasury bond with a unit cost of $1,000.[18] In any case, the savings deposit series tracked a with-cost intermediate Treasury index reasonably well.

All start dates from 1926 to 1975 were examined, but as might be supposed from figure 1, it was necessary to report only retirements in the 1930s and 1960s for this section. These fraught decades suffice to show the comparative success and failure of the four asset allocations considered. A later section will examine the most challenging retirement start date in recent years, January 2000 (Finke et al. 2013), where the result is not yet known but projections can be made.

Table 1 shows the count of failures across each of the four allocations, and the worst case in terms of years sustained, for each of the two test periods. The table also shows the count of what might be called spectacular successes, where the allocation was able to sustain 4-percent withdrawals for 50 years or more. The table makes clear that all the allocations would have failed some of the time. It also shows that retirement

TABLE 1 Failure Rate for More Aggressive vs. More Conservative Allocations

ALLOCATION	1930–1939		1961–1970	
	< 30 YEARS (WORST CASE)	50+ YEARS	<30 YEARS (WORST CASE)	50+ YEARS
100% stocks	1 (21)	8	4 (19)	2
60% stocks/40% bonds	0	6	4 (24)	0
30% stocks/70% bonds	3 (27)	0	7 (25)	0
100% fixed income	8 (20)	0	10 (24)	0

Note. Table 1 uses the stock mutual fund and savings deposit series from McQuarrie (2023) with no sales load deducted. Numbers in parentheses show in years the shortest period that 4-percent withdrawals could be sustained. Rows may not add up to 10 within periods because cases of moderate success–withdrawals sustained for 30 or more years but less than 50 years–are not tabulated.

in the 1960s posed a much stiffer test for a fixed 4-percent withdrawal rate than retirement in the 1930s, with every allocation failing more often in the later period.

The crash of 1929 is the benchmark case for the benefits of diluting an all-stock portfolio with bonds to control drawdown risk, and it is also the type case for sequence-of-returns risk. It is conventional to suppose that either the 30/70 portfolio, or perhaps even a 100-percent allocation to fixed income, will be superior, at least in this one case.

The results do not bear out this expectation. The 100-percent fixed income portfolio showed the worst performance in the 1930s; the 60/40 allocation outperformed the more conservative 30/70 mix; and the all-stock portfolio was the second-best performer.

The crash of 1929 was, in fact, the worst stock market crash in U.S. history, with a real decline at the bottom of 75–85 percent, depending on the metric.[19] But the all-stock portfolio failed only once in this decade, for retirement just after the initial market decline, in January 1930. It is important to understand why. Bengen (1994) noted how deflation helped all allocations during the early 1930s—the initial withdrawal of $4.00 on the hundred would soon have dropped to $3.07 and did not get back to more than $4.00 until 12 years had passed. However, the reason that the all-stock allocation fared so comparatively well is also the briefness of the decline, however steep, and the rapidity of the recovery. From June 1932, the approximate bottom, to June 1933, 12 months later, the SBBI index of stock returns rose by 163 percent. That came too late for the 1930 retiree, but those who retired one year later, in 1931, could sustain 4-percent withdrawals for more than 50 years, and likewise for most of the remaining retirement dates.

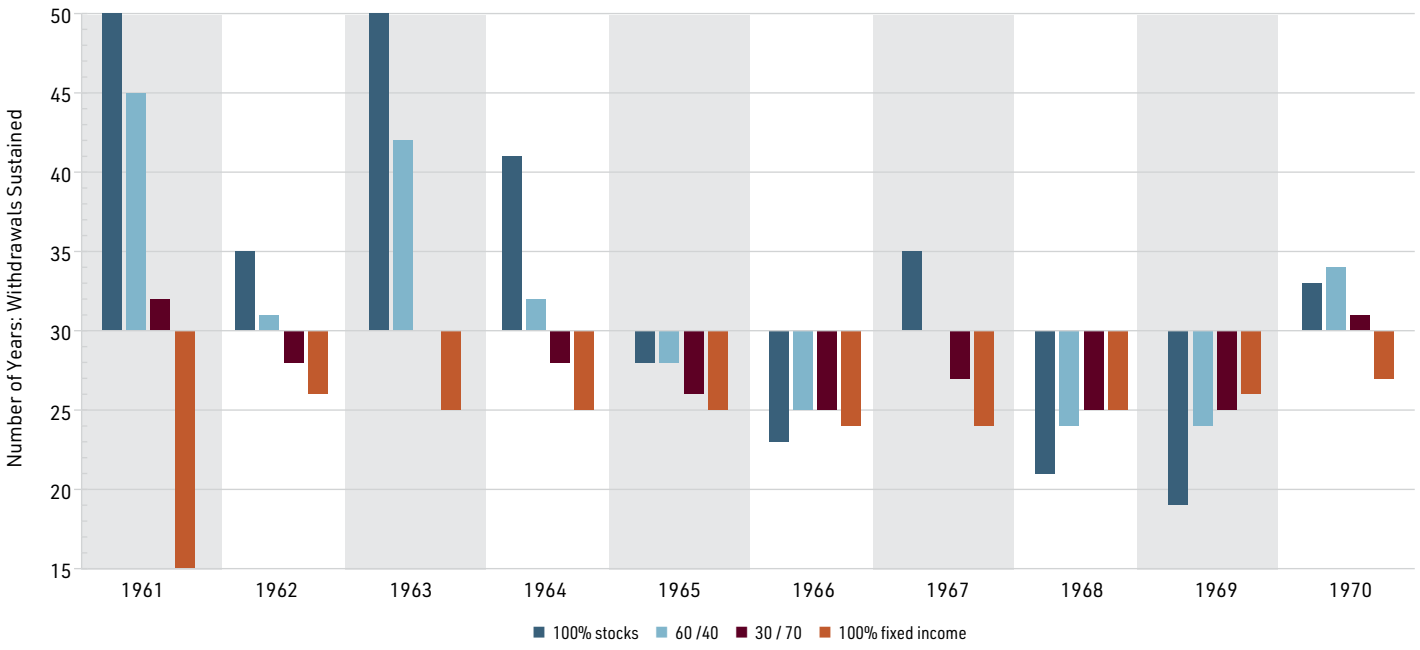
THE CRASH OF 1929 WAS, IN FACT, THE WORST STOCK MARKET CRASH IN U.S. HISTORY, WITH A REAL DECLINE AT THE BOTTOM OF 75–85 PERCENT, DEPENDING ON THE METRIC.

The 100-percent fixed income portfolio fared the worst following the 1929 crash because the interest rate on savings had dropped to 2.5 percent by 1935 and stayed there for decades, even as inflation soared into double digits in the 1940s during and after World War II.

Turning to the 1960s, all the allocations failed multiple times. Nothing availed. Figure 5 shows the magnitude of the shortfalls across the decade. The difficulty that dogged retirement in the 1960s has long been understood. Inflation ran too hot for too long, and stocks same as bonds experienced a prolonged spell of weak real returns. Combining stocks and bonds in different ratios could not overcome the challenge of weak real returns received from both assets.

Stepping back and reviewing the four allocations, conventional wisdom is borne out with respect to the dangers of allocating all a retiree's funds to fixed income. Real rates of return on completely safe investments historically have not met the threshold for sustaining 4-percent inflation-adjusted withdrawals (see figure 3). Peter Bernstein's confidence in the 60/40-allocation also is supported. It had the best or second-best record with respect to failures across those two periods, and the failures that did occur were about as mild or milder as those seen in any other allocation. Surprisingly, the 30/70 allocation did not

FIGURE 5 How Different Asset Allocations Fared for Retirement in the 1960s



Source: Author's calculations.

fulfill expectations that it would fare better than more aggressive allocations in times of stress. Whatever the diversification bonus of combining stocks and bonds, and whatever the reduction in volatility from putting a heavy weight on bonds, this mix allocated too much to the low-return fixed income asset, with its susceptibility to inflation.

Retirement in January 2000

Finke et al. (2013), writing about 10 years after the peak of the 1990s boom, had good reason to question whether the 4-percent rule could survive this new test. Stocks had declined during 2000–2002 and, after a weak recovery, had plunged further during 2007–2008 before bottoming in March 2009.[20] The yield on intermediate Treasuries also was on course to decline below 1 percent in the years following publication of Finke et al. (2013). With stocks sustaining a multi-year drop, and bond yields depressed, renewed skepticism toward the 4-percent rule seemed to be in order.

With the benefit of a dozen additional years of hindsight, it is possible to determine whether any of the asset allocation mixes tested in the 1930s and 1960s are on track to fail to sustain 30 years of withdrawals following retirement in January 2000.

Examining withdrawal success for retirement in 2000 has three additional benefits. First, these events occurred in recent memory. There can be no dismissal of the post-2000 period as "too long ago," or "circumstances were so different." Second, investment technology had made significant strides by 2000. Most notable is the development of the index fund by John Bogle and others. Returns for the Vanguard 500 Index fund are not laboratory data; these are the actual returns, with dividends reinvested, that were available to ordinary investors. Likewise, by 2000 it had become possible to buy an intermediate Treasury fund and receive the after-cost returns on this medium-duration instrument backed by the full faith and credit of the U.S. Treasury. These funds carry no sales load and there is no exchange fee; rebalancing can be exact and does not incur charges.

A third advantage is the availability of assets other than stocks and intermediate Treasuries, most notably, a variety of very-low-risk instruments such as the Treasury money market fund—an equivalent of owning Treasury bills without having to meet the $10,000 threshold. In addition, international stock funds have appeared. In this connection, Anarkulova et al. (2023), using simulations and a deep historical database, argued that there was no need for investors to diversify their stock portfolios with any fixed income at all; a combination of domestic and international stock indexes sufficed to navigate accumulation followed by decumulation. Theirs was a laboratory finding, same as Bengen's; the post-2000 fund results allow a test of real-world after-cost international diversification for the U.S. investor.

Table 2, panel A shows the results so far for the four allocations tested earlier in table 1, using post-2000 fund returns down through 2024, or 25 years in all. Table 2, panel B introduces a selection of pertinent assets and asset mixes that had not been available for testing in earlier periods. The key metric for evaluating these allocations at the 25-year point is the projected number of additional years that withdrawals can be sustained. This can only be an estimate and is made as simple as possible: the end-2024 balance divided by the projected 2025 withdrawal. That quotient is added to the 25 years sustained so far to project whether Bengen's 30-year criterion might be met at the end of 2029.

Taking table 2, panel A first, all four asset mixes are on track to sustain withdrawals to the 30-year criterion. The only allocation having some risk of failure is the 100-percent allocation to stocks. Given a zero net real return during the next five years, it will sustain withdrawals for another 4.8 years, a borderline case. Any real loss in stocks during the period would threaten that success, and conversely, any further gains might avert failure.

TABLE 2 How Different Fund Mixes Fared for Retirement in 2000

ASSET / MIX	VALUE AS OF 2024 (25 YEARS)	PROJECTED ADDITIONAL YEARS	EXPECTED YEARS SUSTAINED IN TOTAL
A: TRADITIONAL			
100% stocks	$3,719	4.8	29.8
60% stocks/40% bonds	$12,997	16.8	41.8
30% stocks/70% bonds	$12,423	16.1	41.1
100% bonds	$8,974	11.6	36.6
B: NEW FUNDS			
Treasury money market	$0	—	[24]
G fund	$3,667	4.7	29.7
Total bond	$7,426	9.6	34.6
Balanced index	$11,664	15.1	40.1
50/50 domestic and international	$2,275	2.9	27.9

Note. Traditional mixes tested using the Vanguard 500 Index fund and the Vanguard Intermediate Treasury fund. New funds use VUSUX for money market, VBTLX for total bond, VBIAX for balanced index, and VFIAX and VTIAX for the domestic-international blend. Projected additional years is the estimated 2025 withdrawal of $772, assuming 3-percent inflation in 2025, divided into the 2024 nominal balances. Starting portfolio value was $10,000.

The 100-percent allocation to intermediate Treasuries was much more successful in this period than the safe asset had been in the 1930s and 1960s,[21] showing an expectation that 36 years of withdrawals in all might be sustained. The 60/40 and 30/70 blends with intermediate Treasuries are the most successful of the set, with each on track to sustain withdrawals for more than 41 years. The reason stock and bond diversification worked so well for the 2000 retiree is straightforward: The correlation between the stock and intermediate Treasury funds was a negative 0.40 across these 25 years, far more favorable than could be expected.[22]

Turning to table 2, panel B , the new assets tested are ordered from most conservative to least. Taking the Treasury bill equivalent first, this completely safe allocation already has failed, running out of money at the end of 2023, after only 24 years, parallel to how savings deposits performed in the 1930s and 1960s (see table 1). Treasury bill rates dropped below 1.0 percent (nominal) in 2009 and stayed below 1.0 percent for the next eight years.

THE INVESTOR GETS THE UPSIDE BENEFIT OF DURATION—THE AVERAGE MATURITY RUNS NEARLY 17 YEARS—WITHOUT ANY DOWNSIDE VOLATILITY RISK.

No advisor should expect a money market fund or other Treasury bill equivalent to sustain real withdrawals as high as 4 percent over three decades. Expectations may run higher, at least on the part of some retirees, for the G fund, available to federal employees as part of their Thrift Savings Plan. This fund pays the average yield on all Treasuries with more than four years remaining to maturity. Like a Treasury bill, there is no possibility of loss.[23] The investor gets the upside benefit of duration—the average maturity runs nearly 17 years—without any downside volatility risk. However, as table 2 shows, a 100-percent allocation to the G fund may not quite reach the 30-year criterion. Almost any shortfall in real return during the next five years will set it back. The problem was the financial repression that followed the 2007–2009 downturn, when G fund nominal returns were depressed to 2.0 percent and below.[24]

The next asset is the total bond fund, now used as the fixed income asset in Vanguard's target retirement funds.[25] It allows the investor to hold the market portfolio in the domestic investment-grade bond space: all bonds with maturity greater than one year, in market weight, comprising more than 10,000 securities, in a single low-cost wrapper.[26] A 100-percent allocation to total bond did quite well for the 2000 retiree, on track to sustain more than 34 years. That's not quite as good as the intermediate Treasury fund, and 60/40 and 30/70 mixes using total bond also were not quite as successful (not tabled). The problem with the blends appears to be the positive correlation of total bond with the stock fund, 0.27 versus a negative 0.40 for intermediate Treasuries, limiting the diversification bonus.

Continuing, Vanguard offers a balanced index fund, which may be thought of as Peter Bernstein's recommended allocation in a convenient, low-cost package: 60 percent allocated to the market portfolio in domestic stocks, 40 percent to the market portfolio in domestic investment-grade bonds. This fund was quite successful, on track to sustain withdrawals for 40 years.

Finally, the bottom row shows results for a 50/50 blend of domestic stocks and international stocks. This allocation is on track to fail at less than 28 years. The relatively poor performance of international stocks in U.S. dollar terms following 2009, and the very high correlation of 0.85 with domestic stocks, may explain the failure.[27]

In summary, perfectly safe investments in nominal terms, under conditions where there is some inflation, are unlikely to return enough to support a real withdrawal rate as high as 4 percent. When stocks plunge, financial repression may cause yields to plunge on completely safe investments as well. When the asset is completely safe against even temporary loss, i.e., no duration risk, there is no capital appreciation to be gained from any fall of yields. Conversely, assets with an intermediate duration, such as total bond and intermediate Treasuries, can benefit when there is a flight to safety. The inadequacy of perfectly safe investments, with respect to sustaining a substantial level of real withdrawals, now has been established on three separate occasions where the timing of retirement was unfortunate, at least for the stock investor of the 1930s, 1960s, and 2000s. Advisors who have very fearful or conservative clients need to hammer this point: It typically is not possible for a completely safe fixed income asset to sustain a withdrawal rate as high as 4-percent real.

More External Validity

There is one additional problem with fixed withdrawal rates. In addition to their dependence on the exact level of real return, the volatility of those returns, and their sequence, the timing of withdrawals also makes a substantial difference to whether withdrawals can be sustained.

Recall that Bengen's withdrawals were taken in an unrealistic manner that also favored success: only once at year end, after the portfolio had been credited with that year's return (Horan 2024). Retirees who expect to live off withdrawals from their portfolios would proceed differently, taking monthly withdrawals beginning January 1 and continuing these on the first of each month. The first withdrawal would be exactly one-twelfth of 4 percent (or 0.3333 percent) of the portfolio value at the end of the prior month and the dollar amount of the second month's withdrawal would be increased by inflation over the trailing month. That procedure would provide an inflation-adjusted stream of monthly income at the indicated annual withdrawal rate, here 4 percent.

This more realistic approach to withdrawals makes a marked difference for the stock investor who retired in 2000. After 300 months, the

TABLE 3 Effect of Monthly Withdrawals for a Retirement in 2000

ASSET MIX (STANDARD DEVIATION)	VALUE AFTER 300 MONTHS	PROJECTED ADDITIONAL MONTHS	PROJECTED TOTAL IN YEARS
100% stock	$1,989	31.9	27.6
Balanced index fund	$10,672	171.0	39.2
Total bond	$7,278	116.6	34.7
Balanced index with 1% fee	$5,061	81.1	31.8
Total bond with inflation +10 bps per month	$4,458	53.0	29.4

all-stock portfolio is now on track to be exhausted in 32 more months, or after less than 28 years (see table 3), validating the fears expressed in Finke et al. (2013).[28] The balanced fund is only slightly impacted, and the effect on total bond is almost invisible, with still about 10 years remaining, same as before. Stepping back, the disparate impact of switching to monthly withdrawals appears to track differences in volatility. The stock series, with a monthly standard deviation of 4.4 percent, suffers with monthly withdrawals, while the other two, with deviations of 2.8 percent and 1.2 percent, respectively, suffer less or not at all. This stands to reason because the stock market was much lower at the end of February 2009 than at the end of 2009. Withdrawals taken at the trough of such downward spikes necessarily are more deleterious than withdrawals taken at the end of the year after some recovery occurred.

The results support the value of volatility damping during decumulation, and therefore of balanced mixes generally. Lest the planner grow too confident that the 4-percent rule can be sustained with modern investment technology when a balanced or other conservative mix is used, the bottom of table 3 shows a re-estimate of the longevity of the 60/40 portfolio after making a small alteration. Monthly returns are reduced by 0.0833 percent to reflect a 1-percent AUM fee, which might be necessary if the client wanted a custom version of the 60/40 blend, or needed hand-holding to stay in the market even with this volatility-damping balanced mix. For the total bond portfolio, a different alteration is considered: Inflation is made to run 10 bps higher per month than the actual record.

These changes have an invidious effect. Where the 60/40 portfolio without fees had been on track to make it 39 years, with a 1-percent annual AUM applied monthly it is now on track to last only 31.75 years. The total bond portfolio, which had been on track to reach 35 years, may now fail at 29.5 years.

Summary

Switching to monthly withdrawals from the volatile stock fund caused a clear failure of the 4- percent rule for retirement in 2000. Adding a standard fee of 1 percent of AUM, billed monthly, almost caused the 60/40 portfolio to fail. And if inflation had run just 10 bps per month higher, owning the total investment-grade bond market would have failed the 2000 retiree.

Summary and Conclusion

Fixed withdrawal rates under field conditions are hazardous to retiree wealth. Any such inflexible system of withdrawals is vulnerable to a tipping point, where returns just a bit lower, or inflation just a bit higher, or a sequence somewhat worse, will lead to premature exhaustion of funds. More generally, laboratory analyses that ignore costs, whether these be the expenses incurred by mutual funds or the fees charged by planners, are likely to be misleading when the goal is to sustain withdrawals for a very long time. Because fixed withdrawal rates are subject to a tipping point, even a slight overestimate of return, consequent to ignoring costs, may show success when a more inclusive account reflective of costs would instead show failure.

This paper marshalled improved historical data for the United States to show that outside the laboratory, the 4-percent rule was not failproof. It went on to explain how fruitless the search for historical support must always be, regardless of reductions in the withdrawal rate to something less than 4 percent. Any substantial withdrawal rate that can be supported on one set of historical data can be shown to fail, if small reductions in return are made to the record, or small increases in inflation are added. Likewise, small increases in volatility or a small rearrangement of the sequence of returns, or even taking withdrawals monthly rather than annually, may suffice to convert a successful record into a failed one. Conversely, a rate so low as to survive all the historical record so far, even after small further reductions in return (Anarkulova et al. 2022), almost certainly will starve retirees of income they could reasonably have enjoyed in the 99 percent of cases apart from the worst-case scenario. That is a failure of decumulation planning.

The problem is not the level set for the withdrawal rate, 4 percent by convention; the problem is fixing the withdrawal rate and making it entirely inflexible over decades. Because fixed withdrawal rates applied against risk portfolios are so sensitive to tipping points, even a little flexibility can go a long way toward sustaining withdrawals.

The paper also called into question the hope that dialing the asset allocation down toward a more conservative mix could rescue an already balanced portfolio at risk of failure. Balanced mutual funds in the 1960s failed to sustain 4-percent withdrawals because the (real) returns on stocks and bonds were both poor—the culprit was the inflation that ravaged both assets.

Finally, the paper showed that international diversification offers no silver bullet. Retirees in 2000 who owned a mix of domestic and international stocks fared worse than those who owned domestic U.S. stocks alone; the dollar returns on international stocks in the following decades were simply too low.

The search for historical data to support some fixed rate of withdrawal is futile. The effort is better spent on testing and comparing approaches to flexible withdrawals, whether using the required minimum distribution schedule in Larson (2022), the virtual annuitization of Waring and Siegel (2015), or the several approaches reviewed in Blanchett (2023).

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ENDNOTES

1. Blanchett (2023) and Horan (2024) provide an overview of the extensive literature on withdrawal strategies.
2. Other researchers have used a wider selection of asset returns to come to similar conclusions, as in Cooley et al. (1998). Bengen (2006; 2025) also considers a wider range of asset mixes.
3. In Bengen and in this paper, retirement in year X means retirement on January 1 of that year, so that calendar year returns can be applied.
4. Inflation values also are taken from the SBBI, which reports the post-1926 average rate of inflation to be about 3 percent.
5. Figure 1 omits the 1976 start year in Bengen, because 50 years of observed returns are not yet available.
6. Bengen assumed a $1-million portfolio split 50/50.
7. The funds used may be found in the appendix to McQuarrie (2023). The source of returns was the Wiesenberger yearbooks (1941–1986), and these give the stock and bond allocations of the funds.
8. Technically, a 1937 start date would have led to exhaustion at 29.56 years with sales load subtracted; this "failure" is ignored.
9. These and later statements about market returns, bond yields, and other characteristics of assets are sourced from the appendix of the SBBI.
10. But only if the withdrawal is made at the end of the year after the return is booked, per the Bengen model. If 4 percent is withdrawn on January 1, the perpetuity will have to pay about 4.17 percent to sustain withdrawals indefinitely.
11. Typically described as "sequence of return risk," or SORR (Estrada 2020).
12. Again, these are returns on the S&P indexes and the five-year Treasury from the SBBI, in a 50/50 blend rebalanced each year.
13. As a reminder, alterations to the arithmetic mean return do not translate one for one to alterations in the annualized (geometric mean) return.
14. At a 50-bps reduction, failure would have occurred at 28.5 years.
15. See Estrada (2020) for a systematic approach to assessing sequence-of-return risk.
16. Although it is correct to describe Retirement Income in broad terms as a 30/70 allocation, it has some distinctive details, holding international as well as domestic stocks and bonds, and allocating about 17 percent of the fixed income portion to a fund containing short-term domestic Treasury Inflation Protected Securities.
17. After 1974, McQuarrie (2023) substitutes an index of money market fund returns, and after 1986, the Vanguard Total Bond index fund.
18. Also, there were no five-year Treasury bonds available before 1934. The SBBI returns used by Bengen for those years were extracted from yield curve interpolations. These can have no external validity because no asset with the indicated yield and maturity could have been purchased. Last, mutual funds owning government bonds, of whatever duration, could not be found back to 1926 and were exceedingly rare before the 1970s.
19. Different indexes show different declines. It matters whether dividends are reinvested, and it matters whether the peak to trough decline is measured over days, weeks, months, or years. Also, because of the 25-percent deflation that occurred over the course of the crash, nominal declines will be larger, as much as 90 percent for some metrics.
20. This author team published several papers on these issues around that time, and I have cited only the one that seemed most relevant. In some of the work the team was more concerned with the low yields on bonds after 2009 than with the stock market declines that followed 1999.
21. It has a longer duration, typically including bonds with from five to 10 years of maturity, or three to 10 years. Interest rates declined from the beginning of the period through 2021, favoring instruments with longer duration.
22. The long-term historical correlation as given in the SBBI has been close to zero.
23. Absent a negative yield across the maturity spectrum, not yet seen in the United States, but approached in some nations outside the United States.
24. Financial repression occurs under a fiat currency when the government suppresses interest rates on those instruments it controls to very low levels.
25. Technically, these funds use a combination of a total international and a total domestic bond fund. But an international bond index fund was not available in early 2000, so therefore it could not be included in this table.
26. However, even the $250-billion Vanguard fund has to engage in sampling. The roughly 10,000 bonds included omit thousands of other bonds included in its benchmark, the Bloomberg Index.
27. This finding does not falsify the Anarkulova et al. (2023) work. These authors, unlike Bengen (1994), did not seek a failproof solution but only a superior strategy, using the criterion of fewer failures.
28. Within the framework of Horan (2024), this test consists of deliberately mismatching the duration of two cash flows: the Bengen tradition, in which withdrawals are taken at the end of the year, versus the commonsense assumption that withdrawals intended to cover living expenses will need to be taken monthly beginning in January. Horan found that when duration is matched, the interval over which withdrawals are taken has no effect, and these results for deliberately mismatched durations do not call that finding into question.

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