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By Massimiliano De Santis, PhD, CFP®

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Abstract

THIS PAPER PROPOSES a dynamic asset and capital allocation strategy for a portfolio designed to sustain regular retirement withdrawals over time. Each withdrawal is modeled as a liability funded by a dedicated stock-and-bond portfolio optimized using Conditional Value-at-Risk (CVaR). Closed-form expressions under lognormal returns allow analytical optimization, yielding horizon-specific allocations that aggregate into a dynamically consistent glidepath. The same tail risk that governs asset allocation is also used to measure the funded status of the portfolio over time and can be used to make small adjustments in spending to manage sequence risk effectively. A Monte Carlo simulation shows that the CVaR-optimized strategy supports a 30-year retirement horizon with a failure rate below 1 percent, with a 4.25-percent initial withdrawal rate. Spending adjustments are modest—less than 4 percent in 95 percent of cases—and occur primarily in early years, effectively managing sequence risk. The results demonstrate that CVaR-based optimization provides a transparent, efficient, and robust approach for sustaining retirement income while adapting risk exposure naturally over time.

Introduction

We propose a capital and asset allocation strategy for a portfolio designed to sustain regular retirement withdrawals over time. The strategy is based on an asset dedication framework that treats every future withdrawal as a liability to be funded. The portfolio dedicated to each liability is composed of stocks and bonds, optimized using tail-risk measures such as Value-at-Risk (VaR) and CVaR. For each liability, the portfolio maximizes expected growth, subject to a chosen tail-risk metric that reflects investor preferences about shortfalls from stated goals. This contrasts with traditional asset dedication where each liability is matched to a fixed income portfolio. Capital is then allocated across liabilities using the same tail-risk metrics and aggregated to obtain a single portfolio.

This design has several advantages relative to traditional retirement portfolio strategies that rely on fixed allocations or age-based glidepaths with constant or guardrail-based spending rules. The first advantage comes from the fact that the same tail-risk metric that informs asset allocation also informs the capital requirements to meet future spending goals (funded status). This information can be tracked over time to suggest spending adjustments to increase the longevity of the portfolio or potential reallocation of capital to other goals.

Second, because liabilities shorten in horizon as time passes, the framework naturally reduces risk exposure over time and generates a dynamically consistent glidepath without relying on heuristics. Finally, the use of objective, quantitative tail-risk measures can complement or even replace traditional risk-assessment questionnaires—an approach especially valuable in employer-sponsored retirement plans where individual preferences may not be observable.

Monte Carlo simulations of the strategy over 10,000 paths confirm its robustness. With a 4.25-percent initial withdrawal rate over 30 years, failure rates (portfolio exhaustion before horizon end) remain below 1 percent under modest dynamic adjustments that never go below 10 percent of the spending target. Adjustments occur in 20–30 percent of paths in the first decade, mitigating sequence risk, but decline thereafter, with 95 percent of outcomes sustaining at least 96 percent of planned spending.

Taken together, these properties make the strategy suitable either as a standalone dynamic income strategy or as part of a broader dynamic allocation approach (De Santis 2023, 2024). By linking portfolio design directly to liabilities and tail-risk control, it provides a disciplined and adaptable foundation for sustainable retirement income.

Comparison to Other Approaches

The proposed strategy complements the literature on asset allocation and spending strategies for retirement. We can group the set of existing strategies into three types:

- Static allocation, constant spending
- Static allocation, dynamic spending
- Dynamic allocation, dynamic spending

Our design belongs to the third group. The strategy is optimal under the chosen preferences and fully dynamic. The use of a tail-risk metric such as CVaR or VaR over individual income liabilities fits naturally with retirement-income preferences. Retirees especially are concerned with the longevity of their portfolios and with keeping downward adjustments to a minimum. In addition, the same metric used to derive the optimal asset allocation is used to compute the funded status of the spending plan and to inform potential spending adjustments. Therefore, our model complements the strategy in De Santis (2024). It also could constitute a floor strategy in the framework of Zwecher (2010). In these approaches, the portfolio is divided into two funds: one dedicated to a committed spending plan with little or no downward adjustment, and a surplus fund invested for growth with the potential to fund future spending increases or other goals. Although De Santis (2024) develops a fully optimal strategy for allocating between the two funds, the paper does not provide a complete strategy for the committed spending component, nor a detailed mechanism for downward spending adjustments. Our design is in part motivated by this gap and is naturally suited to fill it. A related approach is the asset dedication framework as in Huxley and Burns (2018). The main differences are that our design does not rely on exact cash-flow matching and it does not divide the portfolio into arbitrary buckets. Rather, we model the entire income stream as individual liabilities to match.

The second strand of strategies includes dynamic spending strategies with static asset allocation as in Guyton and Klinger (2006), Davis (2010), Kitces (2015), Tharp (2021), and Tharp and Fitzpatrick (2021). All these sources highlight the fact that even simple dynamic spending strategies can increase both the longevity of the portfolio and its overall spending power. However, all these strategies have static allocations. De Santis (2023) shows that a fully dynamic strategy where the asset allocation lever is used in tandem with spending adjustments can increase spending levels and reduce portfolio uncertainty.

The first strand (static allocation, constant spending) includes the seminal paper by Bengen (1994) that uses a balanced 50/50 allocation and derives the 4-percent rule as the spending amount that minimizes the probability of running out of money over 30 years.

The problem with static allocations—including constant allocations or age-based glidepaths—is that they generally are dynamically inconsistent, at least when the goal is stable, sustainable income. For example, the constant mix of stocks and bonds that minimizes the probability of failure over a 30-year horizon is different from the optimal mix over a 25-year horizon, implying that a constant allocation cannot be optimal. A similar argument applies to most static glidepaths unless they are derived from a full optimization (over spending and portfolio choice). However, full glidepath optimization generally requires numerical solutions and cannot provide simple, intuitive, and applicable formulas for allocation and spending.

Our design solves this complex problem by modeling the income stream as a set of individual liabilities and builds a dedicated asset allocation for each liability. The aggregation of this portfolio across buckets gives rise to a natural and dynamically consistent glidepath.

Conceptual Framework

Let the retirement horizon be T years with real (inflation-adjusted) withdrawals each year. Define the set of liabilities:

$$L = \{L_1, L_2, \ldots, L_T\}, \quad L_t = \text{amount due at year } t.$$

We allocate the initial wealth W_0 across T dedicated buckets, one for each liability:

$$\sum_{t=1}^{T} A_t = W_0, \; A_t \geq 0,$$

where A_t is the capital assigned to liability L_t. The liability can be constant, in which case $L_t = x$ for all values of t, but it is not required. Each bucket is invested in a dedicated portfolio of stocks and bonds ($w_t = w_{s,t}, w_{b,t}$). In the following sections, we propose a method for selecting portfolio weights w_t (the asset allocation) and the capital allocation A_t for each horizon-t bucket. The overall allocation of the total portfolio is then derived by aggregating across buckets

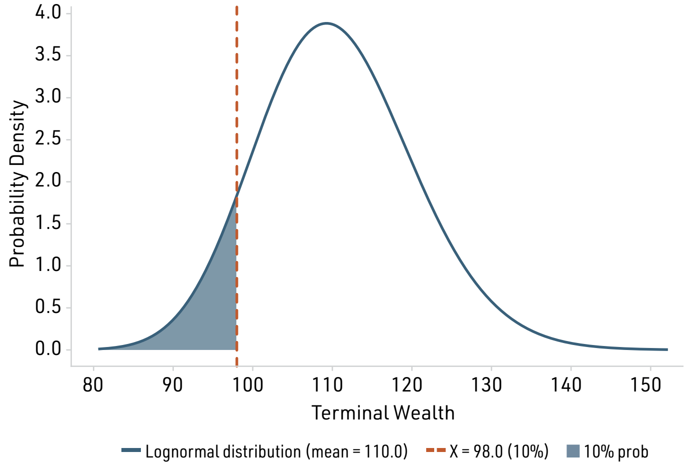
$$\bar{w}_0 = \frac{\sum_{t=1}^{T} A_t \times w_t}{\sum_{t=1}^{T} A_t}.$$

Intuition for the Tail-Risk Based Asset Allocation

We start by deriving the optimal allocation within each bucket and then use the same insight to allocate capital across different buckets.

Consider the five-year bucket as a single goal. For a given portfolio of stocks and bonds ($w_5 = w_{s,5}, w_{b,5}$), the distribution of terminal wealth, TW_5, may look like the one in figure 1. To derive the figure, we assumed an initial investment of \$95, a 20/80 allocation to stocks and bonds, and capital market assumptions that will be detailed below. The level of terminal wealth X corresponding to the 10th percentile is a measure of tail risk. In this case X = \$98, and it is related to what is commonly known as 10-percent VaR or VaR_{10}. Under this distribution, there is a 10-percent chance that spending will be \$98 or less, or there is a

FIGURE 1 Lognormal Distribution with the 10-Percent Value-at-Risk (VaR_{10}) Illustrated



Source: Author's calculations.

10-percent chance that the shortfall will be greater than 2 percent of a $100 spending goal.[1]

It is worth emphasizing that we are looking at the distribution of terminal wealth for one year of income (year 5 in this example). Therefore, there is a one-to-one mapping between terminal wealth at time "t" and spending at time "t." The following statements about terminal wealth, spending shortfall, and VaR are equivalent:

- There is a 90-percent probability that the terminal value will be greater than $98, or a 10-percent chance that the terminal value will be below $98. There is a 10-percent chance that the shortfall will be greater than 2 percent of the stated goal of $100.
- The 10th percentile or VaR_{10} of terminal wealth is $98. This implies that the VaR_{10} of a $105 (1/$95) initial investment is $100, a result we will use later for sizing capital allocations.

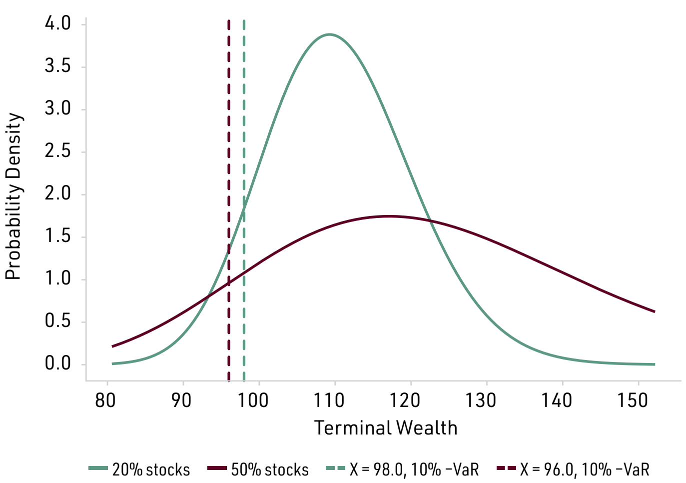
As we change the mix of stocks versus bonds, the shape of the distribution will change and so will the 10th percentile, which can either increase or decrease. Starting from zero equity, the 10th percentile increases as the allocation to stocks rises, reaches a maximum, and then decreases at higher stock allocations. We pick the allocation that maximizes the percentile, which varies by horizon t. We can now define the allocation selection criterion.

Allocation selection criterion

From among the combinations of stocks and bonds $w_t = (w_{s,t}, w_{b,t})$, choose the combination that maximizes a given percentile α of terminal wealth (VaR_α) where α is the chosen percentile or risk level (10 percent in our example).

Figure 2 illustrates this selection by comparing the terminal wealth of the 20/80 allocation with a 50/50 allocation. The 50/50 has a higher mean and median terminal wealth (maroon line); however, it has a lower 10th percentile of terminal wealth ($96 versus $98), so our criterion picks the 20/80 allocation. We can compute the VaR_{10} calculation for all combinations until we find the one with the highest $VaR_{10,}\ w^*_{5,} = (w^*_{s5}, w^*_{b5})$.

FIGURE 2 Comparing the 10-Percent Value at Risk (VaR_{10}) for Two Asset Allocations



Source: Author's calculations.

The process can be repeated for all horizons and liabilities $L_1, \dots L_t, \dots, L_T$. For each bucket t, we choose a constant allocation $w_t = (w_{s,t}, w_{b,t})$ for that bucket to maximize a given percentile (the vertical line in figure 2, or VaR) of terminal wealth:

$$\max_{w_t \in w} \ VaR_\alpha(TW_t),$$

subject to w_t being a valid allocation, e.g., $w_{s,t} + w_{b,t} = 1;\ w_{s,t}, w_{b,t} \geq 0$.

Although the VaR approach is intuitive, it does not consider how bad the terminal wealth shortfall could be within the assumed risk level. To account for the severity of outcomes to the left of the α-percentile, we can compute the expected terminal wealth conditional on being to the left of the α-percentile, known as CVaR. The CVaR answers the question, "When terminal wealth is at or below the VaR_α level, how low will it be?" The answer is that, on average, it will be equal to $CVaR_\alpha$.We have the following selection criterion.

CVaR selection criterion

$$\max_{w_t \in w} \ CVaR_\alpha(TW_t).$$

Because CVaR is a more conservative measure, it yields more conservative allocations relative to the VaR approach. Although we present both metrics in the actual implementation and simulation of the strategy, we will focus on the CVaR approach, which we find more natural for the retirement problem. For an introduction to these two concepts beyond what is outlined here, see Kidd (2012) and the references therein.

In our design, the $CVaR_\alpha$ and VaR_α metrics are not just used for determining asset allocation. They are an integral part of the entire portfolio and spending strategy. In what follows, we are going to use them to do the following:

1. Allocate between stocks and bonds for each horizon bucket t.
2. Estimate the amount of capital needed to sustain the spending liabilities over the entire horizon.
3. Allocate the capital across the horizon buckets $t = 1, 2, \dots, 30$.
4. Track the funded status over time and suggest spending adjustments and reallocate surplus capital.

Analytical Tractability Under Lognormal Returns

If we assume that annual log-returns on stocks and bonds are normally distributed, the VaR and CVaR metrics above have intuitive, closed-form solutions. With the normality of stocks and bonds, the return on a given portfolio is normally distributed:

$$\ln(R_p) \sim N(\mu_p, \sigma_p^2),$$

where R_p are gross returns, and μ_p and σ_p^2 depend on the allocation w. It follows that for horizon t, log terminal wealth also is normally distributed:

$$\ln(W_t) \sim N(t \cdot \mu_p, t \cdot \sigma_p^2).$$

And the α percentile of the distribution is given by

$$\text{VaR}_\alpha(\ln W_t) = t \cdot \mu_p + \sqrt{t} \cdot \sigma_p \Phi^{-1}(\alpha), \quad (1)$$

where Φ^{-1} is the standard normal quantile or score at the chosen percentile level (the NORM.INV function in Excel). The conditional mean below the α percentile takes a similar form:

$$\text{CVaR}_\alpha(\ln W_t) = t \cdot \mu_p - \sqrt{t} \cdot \sigma_p \frac{\phi(z_\alpha)}{\alpha}, \quad (2)$$

where ϕ is the standard normal probability density function (PDF). This eliminates the need for Monte Carlo simulati, making the optimization for each bucket computationally efficient. To find the optimal allocation weights we simply can search over a grid of values for $(w_{s,t}, 1 - w_{s,t})$.

The formulas also are intuitive. Consider the VaR formula first. In maximizing VaR we look for allocations that increase returns (higher equity) subject to a penalty that depends on the left tail of the distribution generated by the portfolio. The penalty increases in the standard deviation of the portfolio σ_p and in the chosen percentile $\Phi^{-1}(a)$. Notice that $\Phi^{-1}(a)$ is negative because it is the score of the standard normal distribution to the left of the mean and increasingly is negative with lower α levels. Higher-equity portfolios increase the mean but also the standard deviation, albeit in a nonlinear fashion because of the imperfect correlation between stocks and bonds. This guarantees that a maximum can be achieved by changing w_t. Finally, consider the effect of the horizon t in determining VaR and CVaR. As the horizon t increases, the first term increases linearly in t while the penalty increases with the square root of t. This implies that the optimal allocation to equities will increase with the horizon.

The CVaR metric has a very similar form, albeit with a heavier penalty because by definition the conditional mean of the standard normal below the α-percentile is smaller than the percentile itself, that is:

$$-\frac{\phi(z_\alpha)}{\alpha} < \Phi^{-1}(\alpha),$$

where the ratio on the left is the conditional mean below the α-percentile, and the function on the right is the α-percentile. Because of this the variance of the portfolio will be penalized more, yielding generally lower optimal allocations to stocks.

HIGHER-EQUITY PORTFOLIOS INCREASE THE MEAN BUT ALSO THE STANDARD DEVIATION, ALBEIT IN A NONLINEAR FASHION BECAUSE OF THE IMPERFECT CORRELATION BETWEEN STOCKS AND BONDS.

Calibration and Bucket Allocation

We derive the allocation to stocks and bonds for all buckets with the following process:

1. Make capital market assumptions.
2. Start with $t = 1$ and compute VaR and CVaR metrics for a grid of equity allocations from 0–100 percent.
3. Select the allocation that maximizes the selected metric.
4. Repeat for all $t = 2, 3, \ldots, 30$.

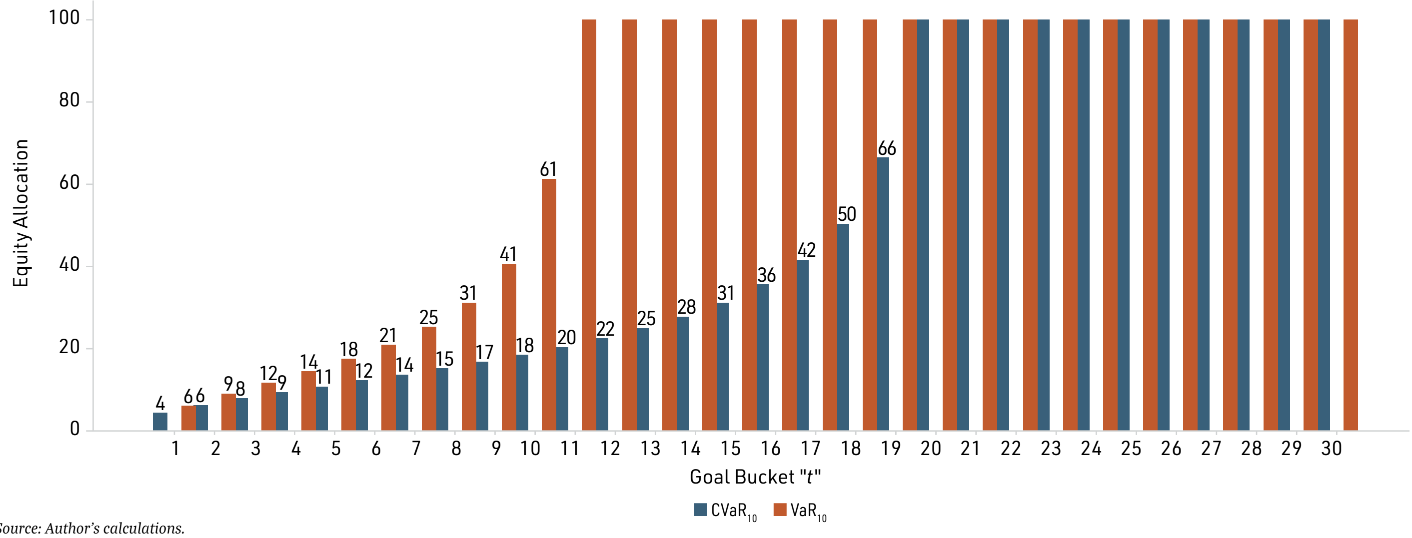
In what follows, we calibrate the parameters for stocks using the Center for Research in Security Prices (CRSP) 1–10 market index, and for bonds using the Bloomberg Government/Credit 1–3-year index. We choose this bond index because of its short duration, which makes it both more representative of fixed income portfolios held by individual investors and a good option for meeting short-term income needs. Returns on this index are available since January 1976, so our sample period is monthly from January 1976 through December 2024. Annualized statistics of returns, adjusted for inflation using the Consumer Price Index (CPI), are reported in table 1.

TABLE 1 Capital Market Assumptions

ANNUALIZED STAT.	STOCKS	BONDS
Mean	8.12%	1.91%
Standard Deviation	15.53%	2.89%
Correlation	18.00%	–
CPI Inflation	3.61%	–

Notes and sources: Real returns. Stocks represented by CRSP 1-10 market index; Bonds represented by the Bloomberg Government/Credit 1-3-year index. Annualized statistics from monthly data.

FIGURE 3 **Time Zero Percentage Allocations to Equities, by Horizon Bucket**



Source: Author's calculations.

For the capital market assumptions above, the procedure yields the equity percentage allocations shown in figure 3 for each bucket. For example, for $t = 5$ years, the VaR optimal allocation is 18 percent versus 12 percent for CVaR. As expected, the optimal allocation to equities increases naturally with the horizon. Notice these are the equity allocations across buckets, at time zero. A year later, the figure would look exactly the same without bucket 30, in year 2, without bucket 29, and so on until the beginning of year 30, when only the first bucket is present.

As figure 3 shows, short-dated liabilities have a conservative allocation, and longer-dated liabilities warrant greater risk taking. Note that the allocations in figure 3 are not a glidepath. The figure simply shows at time zero how the buckets for the different years 1, 2, ..., 30 should be allocated between stocks and bonds. The overall portfolio allocation at time zero will be a weighted average of these individual goal allocations, with weights to be derived below.

However, notice that the allocations in figure 3 will generate a glidepath for the total portfolio. As time passes, the longer-dated goals become closer. So, in year 2, the longest-dated bucket is 29; in year 3 the longest dated bucket is 28, and so on. Therefore, aggregating across buckets will naturally generate a declining asset allocation over time, because we lose long-dated horizons and the allocation at shorter horizons is more conservative.

Optimal Allocations and Capital Sizing

Now that we have derived optimal allocations, how much capital should we allocate to each bucket? Our tail-risk metrics allow us to answer this question. Consider the $CVaR_{10}$ optimization first. Once optimal allocations are chosen, the entire distribution of terminal wealth for $1 (or any amount) of initial investment is known. This allows us to compute $CVaR_a$ for $1 of initial investment for all $t = 1, 2, ..., T$. Thus, if we allocate $\$(1/CVaR_{10,t})$ to bucket t, we expect the $CVaR_{10,t}$ of this investment to be exactly $1. Therefore, we can compute the capital required for all t as:

$$A_t = \frac{L_t}{CVaR_{10,t}}. \qquad (3)$$

Using this formula, total capital required at time 0, W_0, is given by

$$W_0 = \sum_{t=1}^{T} A_t,\ A_t \geq 0.$$

To illustrate, table 2 shows the CVaR and selected percentiles of terminal wealth for a $1/CVaR_{10,t}$ initial investment for each horizon t, using the optimal allocations. Table 2 only reports the data at five-year intervals (a full table is available upon request). By construction, the $CVaR_{10,t}$ of terminal wealth is $1. As expected, the 10th percentile is greater than $1 because the 10th percentile is greater than $1/CVaR_{10,t}$ and so will be the 25th percentile and the median. The final column reports $1/CVaR_{10,t}$. For $t = 1$, its value is greater than one. Because CVaR is a measure of the left tail, to ensure a $CVaR_{10,t}$, we need to invest an amount greater than a dollar.

TABLE 2 **Terminal Wealth Distribution for L_t = \$1 and \$1/ $CVaR_{10,t}$ Initial Investment**

YEAR	CVAR	10TH PCTL	Q25	MEDIAN	1/CVAR
11	1	1.01	1.03	1.05	1.03
5	1	1.04	1.08	1.14	1.00
10	1	1.08	1.15	1.25	0.93
15	1	1.24	1.26	1.46	0.83
20	1	1.39	2.12	3.38	0.71
25	1	1.44	2.31	3.91	0.55
30	1	1.50	2.51	4.45	0.43

Source: Author's calculations.

However, as the horizon increases, the entire wealth distribution moves to the right and the capital required to obtain a $CVaR_{10,t}$ = \$1 declines. For year 30, the capital required for a \$1 liability is \$0.43.

With this approach we can compute the total wealth required for a \$1 annual liability as

$$TW_{CVaR}(\$1, \ldots, \$1) = \sum_{t=1}^{T} \frac{1}{CVaR_{10,t}} = \$23.54,$$

which sums to \$23.54 using our parameterization. We can use this value to find the capital required to fund any spending amount. Suppose we are interested in funding \$40,000 in annual spending; the capital requirement $W_0 = 40{,}000 \times 23.54 = \$941{,}600$. The amounts to be allocated to different buckets are given by the values in the final column of table 2. From the table, $A_1 = 1.03 \times 40{,}000 = \$41{,}200$, or 4.4 percent of W_0.

The VaR metric can be applied similarly. We can compute the total wealth required for a \$1 liability as

$$TW_{VaR}(\$1, \ldots, \$1) = \sum_{t=1}^{T} \frac{1}{VaR_{10,t}} = \$19.9,$$

under our parameterization. So, using this metric, the goal is equivalent to a $W_0 = \$794{,}000$. The amounts to allocate to different buckets are

$$A_t = \frac{L_t}{VaR_{10,t}}. \quad (4)$$

Under our parameterization, $A_1 = \$40{,}652$, which is 5.12 percent of the total value of \$794,000. Relative to the CVaR model, VaR is a lower capital requirement and relies on a more aggressive asset allocation. The cost of this will be more frequent or larger spending adjustments over time, so this approach may be better suited to a more risk tolerant and less capitalized investor.

Performance Monitoring Using the Metrics

The total wealth requirements above can be computed for all years in the retirement horizon. Each future year "τ," the remaining total wealth requirement under the metrics can be computed as

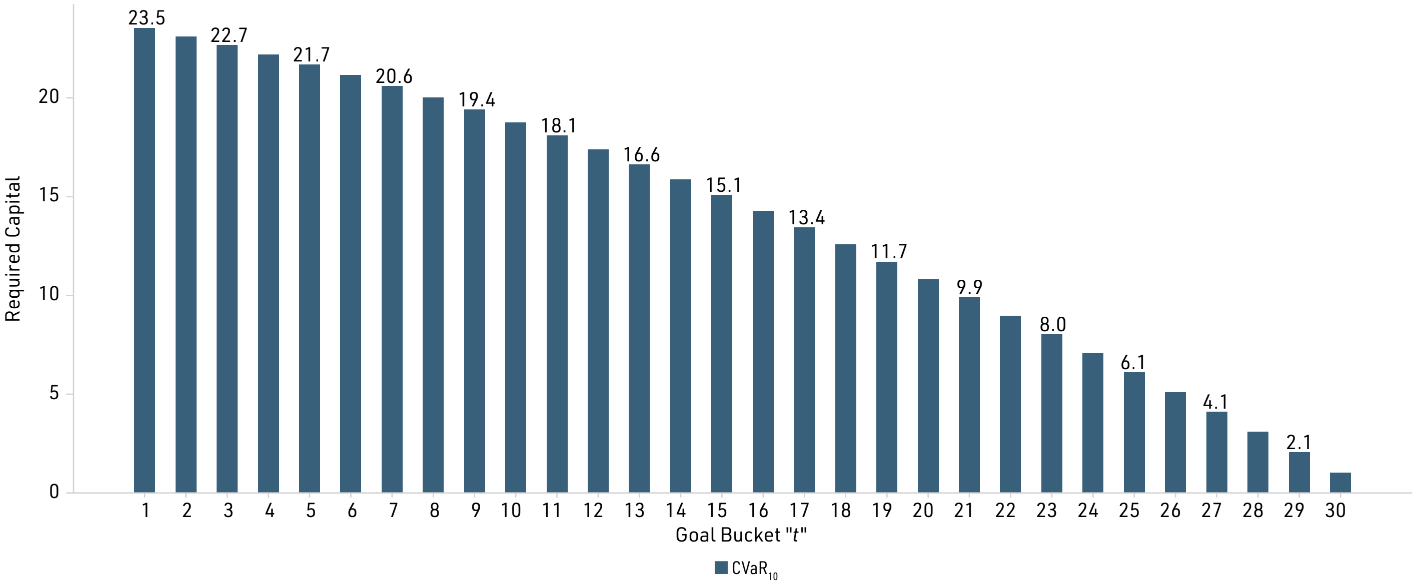
$$\bar{W}(\tau) = \sum_{t=1}^{T-\tau} \frac{L_t}{CVaR_{10,t}} \quad (5)$$

$$\bar{W}(\tau) = \sum_{t=1}^{T-\tau} \frac{L_t}{VaR_{10,t}}, \quad (6)$$

where the bar over W, or $\bar{W}$, indicates that this is a required wealth level according to the metrics, and it can differ in practice from realized wealth, $W\tau$. Over time, the sums above lose the longer horizon terms (30, 29, ...) and the requirements decline in value. These equations can be used to monitor the level of funding over time.

If the actual wealth $W_\tau \geq \bar{W}(\tau)$, the plan needs no adjustments, and the level of spending can continue. In fact, surplus is created and can be retained to protect from future spending declines or reallocated to other goals. If, however, $W_\tau < \bar{W}(\tau)$, there is not enough capital to maintain the same spending level. Spending can be adjusted downward, or additional capital is required to keep the same level of spending—for example, from a surplus bucket, as in De Santis (2024). Notice that $\bar{W}$ values can be easily computed because we have the full sum available at time zero. They do not need to be recomputed unless the parameter calibration changes. For our parameterization, the evolution of $\bar{W}(\tau)$ under the $CVaR_{10}$ metric is shown in figure 4.

FIGURE 4 Evolution of Required Capital $\bar{W}(\tau)$, by Horizon, \$1 Liability



Source: Author's calculations.

Aggregate Portfolio and Glidepath

Having derived optimal weights w_t^* and capital allocations A_t, the overall portfolio allocation at time $t = 0$ is given by the aggregation of all the buckets:

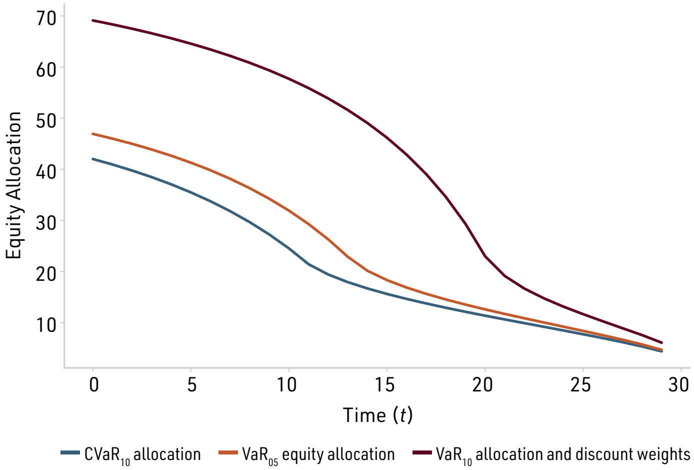
$$\bar{w}_0 = \frac{\sum_{t=0}^{T} A_t \times w_t^*}{\sum_{t=0}^{T} A_t}. \tag{7}$$

For each year τ after that, we apply the same process as above, realizing that now there are fewer future periods. So, after a year, $\tau = 1$, the buckets go from one to 29; at $\tau = 2$, they go from one to 28, and so on. Thus, updating the equity allocation for the portfolio described by equation (7) over time yields:

$$\bar{w}_\tau = \frac{\sum_{t=1}^{T-\tau} A_t \times w_t^*}{\sum_{t=1}^{T-\tau} A_t},$$

which declines as short-horizon, bond-heavy buckets dominate the remaining wealth. Thus, the methodology naturally gives rise to a declining glidepath of portfolio allocation. The glidepath is illustrated in figure 5 for selected VaR and CVaR metrics and corresponding weighting schemes.

FIGURE 5 Evolution of Allocation Along a Glidepath



Source: Author's calculations.

Consistent with figure 3 and the capital sizing results, the VaR_{10} glidepath is substantially more aggressive than the $CVaR_{10}$. Also shown in the figure is the VaR_5, with very similar results to the $CVaR_{10}$ glidepath.

A Dynamic Spending Strategy and Simulation

Our design suggests a simple dynamic strategy for spending and capital allocation in which, at each point in time, we use the CVaR metric to adjust asset allocation and, depending on funded status, spending. We illustrate the approach using a Monte Carlo simulation. In the simulation, we model the portfolio returns using a bivariate lognormal distribution to reflect the returns of stocks and bonds, using the parameterization of table 1. We assume the glidepath generated by the $CVaR_{10}$ strategy, which dynamically adjusts stock and bond allocations over a 30-year horizon. We initialize the portfolio with a wealth of $W_0 = 23.54$, the amount we found is required to fund $1 of spending over the 30-year horizon under $CVaR_{10}$. To analyze the sustainability of the dynamic strategy, we simulate 10,000 histories of portfolio returns over 30 years.

Over each future year τ, the plan is to spend $1, contingent on an adjustment mechanism that evaluates the remaining wealth (W_τ) relative to the weighted sum of remaining payments, $\bar{W}(\tau)$ from equation (5).

Spending over time is based on the equation:

$$S_\tau = max\left(S_{min}, f \cdot \frac{W_\tau}{\bar{W}(\tau)} + (1 - f) \cdot 1\right), \tag{8}$$

subject to the condition that $S_\tau \leq \$1$, that is, spending can be adjusted downward if the funded status is too low and equals $1 otherwise.

In equation (8), $S_{\min}$ is a minimum level of spending that can be chosen based on flexibility or simulations. The ratio

$$\frac{W_\tau}{\bar{W}(\tau)}$$

is the CVaR-based funded status over time, and the denominator $\bar{W}(\tau)$ is the total capital requirement illustrated in figure 4. If this ratio is less than one, spending is reduced. However, not all of the decline in funded status of the portfolio is applied to current spending, and the coefficient f is used to control that. The greater f the larger the downward adjustment. Finally, if S_τ is ever greater than one based on equation (8), we clip it and simply set spending to $1.

Notice the simplicity of the strategy over time. Allocation evolves according to the glidepath in figure 5. To evaluate whether the portfolio is on track to sustain the spending goal, we just compare the current portfolio value with the level presented in figure 4, computing the ratio

$$\frac{W_\tau}{\bar{W}(\tau)}.$$

Simulation Results

The first question we can ask from the simulation is about failure rates. The $CVaR_{10}$ of each bucket is $1 by construction, but there still is a chance that terminal wealth at each horizon is lower than $1. Although the overall portfolio allocation is relatively conservative (see figure 5), without any adjustment the probability of failure is non-zero. So, as a baseline, we ran a simulation setting $S_{\min}$ to 1 and f to zero (no adjustment). In this simulation, the overall failure rate (wealth is exhausted before $T = 30$), is 2.27 percent (227 times out of 10,000 simulated histories). The longevity of the portfolio in a failed run is at least 23 years. These already are acceptable results and speak to the robustness of the strategy. Of note is that \$1 of spending corresponds to 4.25 percent of initial spending, or a year with W_0 = \$941,600 in our earlier example.

These good baseline results also mean that we can improve the failure probability by making small adjustments. In light of this, we select $S_{\min} = 0.9$ and $f = 0.5$. With these parameters, spending can never be below 90 percent of the original plan. The $f = 0.5$ assumption means that we only adjust downward halfway from the proposed adjustment based on funded status alone. The failure rate with these assumptions drops below 1 percent (0.84 percent) and the longevity is now at least 25 years. As to be expected, most adjustments using this strategy occur in the early years, with adjustments needed about 20–30 percent of the time during the first decade. The adjustment frequency declines after that and falls below 10 percent by year 16. The high frequency in the early years is another manifestation of what often is referred to as sequence risk, which affects the longevity of the portfolio, in particular during the first 10 years of a retirement horizon. Spending adjustments are the way in which sequence risk is actively managed in the strategy. Figure 6 shows the variation of funded status at selected points in time, by plotting its frequency distribution.

The funded level fans out over time, as uncertainty increases with the horizon. However, the likelihood of the funded level being less than one declines over time as the distribution, although wider, skews toward the right. For the 20-year distribution, the likelihood that funded status is less than one and spending needs to be adjusted is only 6 percent.

Table 3 reports the percentiles of the spending distribution over time for year 1 and at five-year intervals after that. As table 3 shows, spending is never below 90 percent of the initial goal, given the $S_{\min}$ requirement. This low level is reached less than 5 percent of the time. The lowest 5th percentile is 94 percent of the goal, at year 10. The 5th percentiles across all years and simulations (final row) is 96 percent of the goal. This means that 95 percent of the time, the adjustment is less than 4 percent of the original target.

TABLE 3 Percentiles of the Spending Distribution Over Time

YEAR	MIN	P(1)	P(5)	P(10)	P(25)	MEDIAN
1	0.97	0.98	0.99	0.99	1.00	1.00
5	0.90	0.93	0.95	0.97	0.99	1.00
10	0.90	0.91	0.94	0.96	1.00	1.00
15	0.90	0.90	0.95	0.98	1.00	1.00
20	0.90	0.90	0.98	1.00	1.00	1.00
25	0.90	0.90	1.00	1.00	1.00	1.00
30	0.90	0.90	1.00	1.00	1.00	1.00
Total	0.90	0.90	0.96	0.98	1.00	1.00

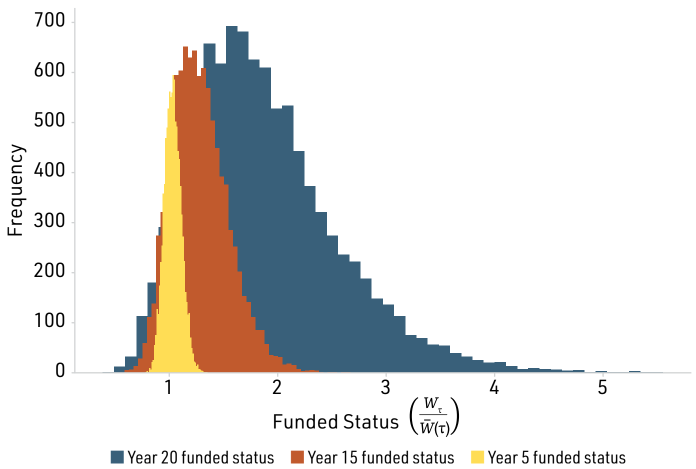
Source: Author's calculations.

Surplus Wealth

The focus of the dynamic spending strategy is to limit downward spending adjustments, and equation (8) does not allow for increased spending or reallocating capital to other goals. This is because we view this portfolio as primarily for generating sustainable income. Obviously, however, the funded status can be used as a gauge to increase spending or build surplus capital for other goals. The distribution of ending wealth in the simulation, shown in figure 7, shows that the potential for increased spending is there. One way to increase spending is to relax the condition that $S_\tau \leq 1$ in equation (8) or use the additional capital for other goals. We don't pursue these possibilities here, but we refer to De Santis (2024) or Zwecher (2010) and discussed how our design complements their strategy earlier in the paper.

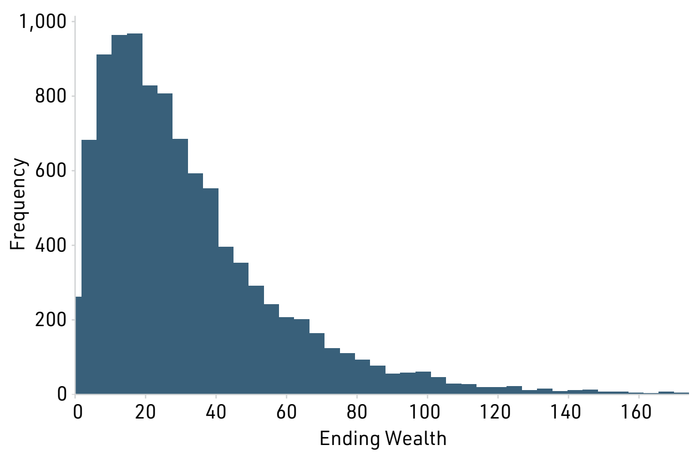
Notably, the optimal allocation is achieved with a relatively conservative allocation to stocks over time, confirming findings from other studies that sustainable income benefits from moderate exposure to stocks, and aggressive allocations may increase the risk of ruin (see, for example, Davis 2010). The $CVaR_{10}$ glidepaths start just below 45 percent at year zero and decline over time as shown in figure 5.

FIGURE 6 Distribution of Funded Status at Selected Years



Source: Author's calculations.

FIGURE 7 Distribution of Ending Wealth in the Dynamic Simulation



Source: Author's calculations.

Summary and Conclusions

We propose a fully dynamic strategy for asset allocation and spending from a retirement portfolio, emphasizing sustainability of regular withdrawals over a multi-decade horizon. The strategy has these main features:

GOAL ALIGNMENT. Each withdrawal is explicitly funded and optimized for its horizon.

TAIL-RISK CONTROL. The CVaR-based optimization ensures robustness against adverse sequences of returns.

ANALYTICAL SIMPLICITY. Under lognormal assumptions, CVaR, allocations, and adjustments are easily computed, avoiding simulation.

BUILT-IN PERFORMANCE MONITORING. Tail-risk metrics track funded status over time, informing spending adjustments to manage sequence risk and increase portfolio longevity.

Monte Carlo simulations demonstrate robust performance. With a 4.25-percent initial withdrawal rate, baseline failure rates (portfolio exhaustion before 30 years) remain under 3 percent, dropping below 1 percent with modest dynamic adjustments, e.g., capping cuts at 90 percent of plan and applying half the implied reduction. Adjustments are most frequent early on (20–30 percent of paths in the first decade), addressing sequence risk, while 95 percent of outcomes sustain at least 96 percent of the target spending level across simulations.

In conclusion, this tail-risk based framework offers a disciplined, implementable tool for retirement-income management. It bridges gaps in existing dynamic strategies by providing a complete, analytically tractable solution for committed spending components, with built-in safeguards to improve portfolio longevity. Future extensions could incorporate multi-asset classes or behavioral preferences, further enhancing applicability in advisor practice or plan design.

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Massimiliano De Santis, PhD, CFP®, is clinical assistant professor of finance and assistant director of the Langston Wealth Management Center, McCombs School of Business at the University of Texas, Austin. Contact him at massi.desantis@mccombs.edu.

ENDNOTE

1. Although the VaR and CVaR metrics usually refer to losses or shortfalls, we will continue to use terminal wealth TW, and refer to our measure of risk, the percentile of terminal wealth, as VaR. Our proposed optimization would yield the same results in terms of shortfalls or losses, and so we prefer to work in terms of the more intuitive distribution of terminal wealth.

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INVESTMENTS & WEALTH INSTITUTE®

5619 DTC Parkway, Suite 600
Greenwood Village, CO 80111
Phone: +1 303-770-3377
Fax: +1 303-770-1812
www.investmentsandwealth.org